

Acoustic Illusion of a Rigid Object for a Moving Emitter via Equivalent Surface Control

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We report a three-dimensional acoustic illusion of a rigid object illuminated by an emitter moving along an arbitrary trajectory. The object is absent: control sources are arranged around its location, the closed surface they define being termed the equivalent surface. Driven by the field the object would have scattered onto this surface, the sources radiate, by the Kirchhoff–Helmholtz representation, that same field beyond it. The control signals follow in closed form from a time-dependent physical optics formulation that we derive for a moving emitter, in which the motion enters analytically through the acoustic travel times, so that the scattering problem need not be re-solved at each emitter position, nor the propagation domain discretized. For objects too large to enclose, the signals are mapped onto an internal surface, an option whose advantages and penalties are discussed. Numerical simulations validate the method in both configurations.

I. INTRODUCTION

Acoustic illusion refers to the synthesis of a prescribed acoustic field such that a receiver array or imaging system perceives an object that is not physically present. The acoustic signature to be reproduced may be either the echoes a target object scatters when probed by an external sound or the noise it radiates of its own accord. The applications are broad. In nondestructive testing, defect signatures can be emulated for the calibration of inspection procedures [1, 2]. In underwater acoustics, entire scenes can be rendered for sonar calibration and operator training, reducing dependence on costly at-sea trials [3, 4]. The defense counterpart is sonar deception, in which an adversary’s sonar is presented with the response of a target of chosen geometry or motion [5].

Active illusions have been pursued by several approaches. In three-dimensional wavefield engineering, control sources are usually placed at arbitrary positions, and their driving signals are obtained by numerical optimization, solving an inverse problem that matches a prescribed target field at a set of control points. The inversion is typically performed in the frequency domain, using regularized least squares with measured or analytical transfer functions [6–8], and underlies applications ranging from personal audio, sound zoning, and cross-talk cancellation, to stealth and large-scale outdoor sound field control [9–13]. In one or two dimensions, physics-based control may bypass numerical minimization, producing illusions of spatial or spatio-temporal voids [14, 15].

Another approach is based on integral representations, specifically the Kirchhoff–Helmholtz (KH) theorem [16, 17]. Following from Green’s second identity, the theorem states that if the acoustic pressure and its normal derivative are known on a closed surface, the field is determined at every point of the source-free region, while the integral vanishes in the complementary one. The boundary quantities may be time varying, the representation then holding in the time domain.

Two branches follow, according to which side the sources lie on. The interior form, with sources outside the surface and the field reconstructed within, underlies Huygens’ principle. In the exterior form, adopted here and illustrated in Fig. 1(a), the sources are enclosed and the field is reconstructed outside,

the radiation condition ensuring that the contribution from the surface at infinity vanishes.

The illusion follows by reading the theorem in reverse. Rather than evaluating a field from known boundary data, the scattered field that the emulated object would produce is computed on a chosen enclosing surface and used to drive control sources distributed over it, as illustrated in Fig. 1(b). By the exterior form of the theorem, the array then radiates precisely the field the object would have scattered, although the object is absent. The surface is acoustically transparent and exists only as the locus of the sources; it is therefore termed an equivalent surface, or control surface, and its shape is unconstrained provided it is closed and encloses the object.

This approach underlies the immersive boundary condition framework [18], in which sources on a closed surface are driven so as to reproduce the response of a virtual scatterer. The transfer functions relating the surface sources to the field are obtained either from full-waveform finite-element simulations of the emulated object [19], or retrieved directly from measurements of a physical specimen [20]. Because these experiments are performed in a laboratory of finite extent, the emulation is accompanied by a second task, namely accounting for the reflections of enclosing boundaries.

Here we address an emitter in motion in an unbounded three-dimensional medium. Its signal and position are taken as available while it moves, the trajectory being either prescribed in advance or estimated in real time by an independent tracking algorithm. In principle, the control signals may be evaluated by any method capable of predicting the scattered field of the object. The finite element method (FEM) [21] and the finite-difference time-domain (FDTD) method [22] discretize the propagation domain and require truncation by absorbing boundaries, while the boundary element method (BEM) [23] discretizes only the object surface, but couples every surface element to every other, so that a dense linear system must be solved.

We adopt instead physical optics (PO), which approximates the surface field directly, taking it to be that of a locally flat rigid reflector over the illuminated portion and zero elsewhere, so that the scattered field follows by direct integration [24–27]. Shadowing and edge diffraction are thereby reproduced approximately, the shadow forming where the scattered field

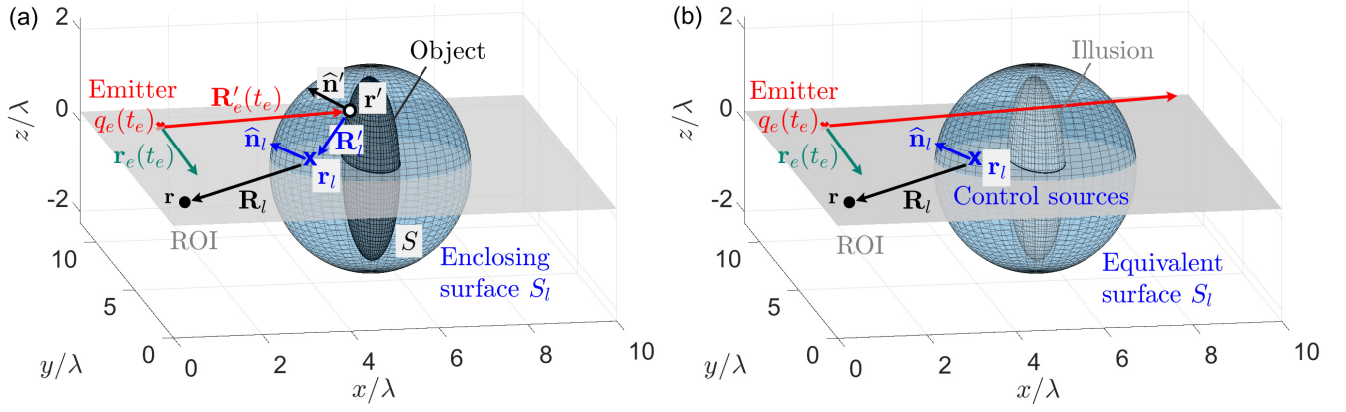


FIG. 1: Model schematics. (a) Kirchhoff-Helmholtz representation. An emitter q_e outside the enclosing surface S_l illuminates a rigid object, and the scattered field at any exterior point $\mathbf{r}$ follows from the scattered pressure and normal velocity at all points $\mathbf{r}_l$ on S_l . (b) Illusion based on the representation. The object is absent and S_l is an equivalent surface carrying monopole and dipole control sources driven by these quantities, reproducing the scattered field in the region of interest (ROI).

cancels the incident one. No system is solved and no propagation domain is meshed. The technique works best for objects with local radii of curvature large compared to the wavelength.

Since the problem considered here is transient, owing both to the emitter's motion and to the possible transiency of its signal, the established frequency-domain form of PO cannot be applied directly. A time-domain version of the method is therefore required. We recently derived such a formulation for a moving emitter [28], referred to here as time-dependent physical optics (TDPO), whose main principles are recapped in Sec. II B.

Moving sources can also be accommodated in time-domain FEM, by redistributing the source forcing among the nodes as it advances or updating the mesh [29], in BEM, by evaluating the incident field on the object boundary at the retarded emission times [30], and in FDTD, by interpolating the source onto the fixed grid [31]. These methods, however, still operate on a discretized domain or boundary, whereas in TDPO the scattered field follows by direct integration, at a far lower computational cost.

Combining TDPO with the time-domain KH representation, the control signals on the equivalent surface follow in closed form for an emitter moving along an arbitrary trajectory, with no discretization of the propagation domain. We first develop and validate this approach in the main configuration, in which the equivalent surface encloses the emulated object (Sec. II). As enclosure may become impractical for large objects, we then consider a smaller equivalent surface that does not enclose it (Sec. III), and discuss the benefits and limitations of the two configurations (Sec. IV).

II. ILLUSION VIA AN ENCLOSING CONTROL SURFACE

The setup is defined in Fig. 1. The emulated object is perfectly rigid, stationary, and defined by the bounding surface S . The formulation applies to an arbitrary object geometry; a simple convex shape is depicted in Fig. 1 for clarity, whereas

the numerical examples below use a more general one. The surrounding medium is homogeneous, isotropic and lossless, with mass density ρ and sound speed c , and is unbounded.

The illuminating emitter is a monopole of strength $q_e(t_e)$ (in kg/s^2), the time derivative of its mass injection rate, following a trajectory $\mathbf{r}_e(t_e)$, where t_e denotes the emission time. In the absence of the object it produces the incident field $p_{\text{inc}}(\mathbf{r}, t)$ at an observation point $\mathbf{r}$ and observation time t ; were the object present, the incident field would induce a scattered field $p_{\text{scat}}(\mathbf{r}, t)$, and the total field would be $p_{\text{tot}} = p_{\text{inc}} + p_{\text{scat}}$. The region of interest (ROI), on which the illusion is to be generated, may be any subset of the three-dimensional space; for clarity of presentation it is taken here as the plane $z = 0$, with the emitter moving within it.

The control sources are distributed over the equivalent surface S_l , assumed closed and sufficiently smooth but otherwise arbitrary, provided that S lies inside the volume it bounds; a spherical S_l is used in Fig. 1. They are monopoles and dipoles of strengths q and F , the dipoles oriented along the outward normal $\hat{\mathbf{n}}_l$. The theorem requires both layers on S_l , but not necessarily at the same points; they are collocated here for convenience. The continuous description is an idealization: in practice the sources form a discrete array, sampled according to the wavelength λ .

The task is to determine the control signals q and F such that the field radiated by the control sources reproduces p_{scat} throughout the ROI. We first consider the established case of a stationary emitter radiating a harmonic signal, for which a steady state is reached and the problem admits a frequency-domain formulation, with the scattered field evaluated by PO (Sec. II A). The time-domain formulation for a moving emitter follows in Sec. II B.

A. Frequency domain formulation

In this section the emitter is stationary, $\mathbf{r}_e(t_e) = \mathbf{r}_e$, and radiates a harmonic signal of complex strength $\tilde{q}_e(\omega)$ at an-

gular frequency ω , so that a steady state is reached. We assume time-harmonic dependence $e^{j\omega t}$, suppressed throughout for brevity. Were the object present, as in Fig. 1(a), its scattered field on S_l would have pressure $\tilde{p}_{\text{scat}}$ and outward normal velocity $\tilde{v}_{\text{scat}} = \tilde{\mathbf{v}}_{\text{scat}} \cdot \hat{\mathbf{n}}_l$, the latter scaled as $\tilde{u}_{\text{scat}} = j\omega\rho\tilde{v}_{\text{scat}}$, so that $\tilde{u}_{\text{scat}} = -\partial_{n_l}\tilde{p}_{\text{scat}}$. With $\mathbf{R}_l = \mathbf{r} - \mathbf{r}_l$ the vector from a control source to the observation point and $R_l = |\mathbf{R}_l|$, the scattered field at any $\mathbf{r}$ exterior to S_l follows from the KH representation as

$$\tilde{p}_{\text{scat}}(\mathbf{r}; \omega) = \int_{S_l} \left[G^{p|q}(R_l; \omega) \tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega) + G^{p|F}(R_l; \omega) \tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega) \right] dS_l, \quad (1)$$

the same integral vanishing for $\mathbf{r}$ interior to S_l .

In the illusion, Fig. 1(b), the monopole and dipole layers of control sources on S_l are thus driven, respectively, by $\tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega)$ and $\tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega)$. Since (1) integrates over S_l , both are surface densities, in $\text{kg}/(\text{m}^2 \text{s}^2)$ and Pa. A realizable array is discrete, its L sources at $\mathbf{r}_l^{(i)}$ carrying the lumped strengths

$$\tilde{q}^{(i)}(\omega) = \tilde{u}_{\text{scat}}(\mathbf{r}_l^{(i)}; \omega) dS_l^{(i)}, \quad (2a)$$

$$\tilde{F}^{(i)}(\omega) = \tilde{p}_{\text{scat}}(\mathbf{r}_l^{(i)}; \omega) dS_l^{(i)}, \quad (2b)$$

in kg/s^2 and N, respectively. The monopole is a point volume source, $\tilde{q}^{(i)}$ being $j\omega\rho$ times its volume velocity $\tilde{Q}^{(i)}(\omega)$, and the dipole is a point force $\tilde{\mathbf{F}}^{(i)} = \tilde{F}^{(i)} \hat{\mathbf{n}}_l$ directed along the outward normal. The field radiated by the discrete array, the controlled counterpart of (1), then approximates the scattered field as $\tilde{p}_{\text{scat}}(\mathbf{r}; \omega) \approx \sum_{i=1}^L [G^{p|q}(R_l^{(i)}; \omega) \tilde{q}^{(i)}(\omega) + G^{p|F}(R_l^{(i)}; \omega) \tilde{F}^{(i)}(\omega)]$. Here, $G^{p|q}$ and $G^{p|F}$ denote the corresponding Green's function kernels, given by

$$G^{p|q}(R_l; \omega) = G(R_l; \omega), \quad (3a)$$

$$G^{p|F}(R_l; \omega) = G(R_l; \omega) \alpha_l \hat{\mathbf{R}}_l \cdot \hat{\mathbf{n}}_l, \quad (3b)$$

where $\alpha_l = jk + 1/R_l$, with $k = \omega/c$ the wavenumber, $G^{p|F} = \partial_{n_l}G = \nabla_{\mathbf{r}_l}G \cdot \hat{\mathbf{n}}_l$, and $\hat{\mathbf{R}}_l = \mathbf{R}_l/R_l$. The function $G(r; \omega)$ is the three-dimensional free-space Green's function, which for a general source–observer distance r is defined as [17]

$$G(r; \omega) = \frac{e^{-jkr}}{4\pi r}. \quad (4)$$

The kernels $G^{p|q}$ and $G^{p|F}$ thus account for propagation from S_l to the ROI, so that the reconstruction (1) follows once the control signals (2) are determined. As stated in Sec. I, these follow from $\tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega)$ and $\tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega)$, which can in principle be evaluated by BEM or by grid-based methods such as FEM and FDTD, and are obtained here from the PO approximation.

By the PO approach, the scattered pressure at a point $\mathbf{r}_l$ on S_l is given by the contribution of the lit portion of the object

surface S , defined by $\mathbf{R}'_e \cdot \hat{\mathbf{n}}' < 0$, as [24–27]

$$\tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega) = \int_{S_{\text{lit}}} 2\tilde{q}_e(\omega) G(R'_e; \omega) G(R'_l; \omega) \alpha'_l \hat{\mathbf{R}}'_l \cdot \hat{\mathbf{n}}' ds'. \quad (5)$$

Here, $\hat{\mathbf{n}}'$ is the outward unit normal at the point $\mathbf{r}' \in S$, and

$$\mathbf{R}'_e = \mathbf{r}' - \mathbf{r}_e, \quad \mathbf{R}'_l = \mathbf{r}_l - \mathbf{r}', \quad (6)$$

are the vectors from the emitter to S and from S to S_l , with $R'_e = |\mathbf{R}'_e|$, $R'_l = |\mathbf{R}'_l|$ and $\hat{\mathbf{R}}'_l = \mathbf{R}'_l/R'_l$. Both are time-invariant here, the emitter, the object and S_l all being stationary. Further, α'_l is obtained by substituting $R_l \rightarrow R'_l$ in α_l in (3b), and ds' is the differential surface element. The Green's functions $G(R'_e; \omega)$ and $G(R'_l; \omega)$ follow from (4). For numerical evaluation, S is discretized into elemental cells and the integral over S_{lit} is approximated by a summation. Then, $\tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega)$ can be obtained analytically by differentiating the integrand of (5) with respect to $\mathbf{r}_l$, which yields a lengthier kernel. It is more conveniently evaluated by a finite difference between the pressures on the offset surfaces $S_l^\pm$, defined by $\mathbf{r}_l \pm h \hat{\mathbf{n}}_l$, each obtained from (5).

To demonstrate the control algorithm described by (1)–(6), we consider the geometrically complex object composed of two intersecting cylinders shown in Fig. 2(a). This object is not physically present, and the task of the control system in Fig. 2(b) is to emulate its presence by generating the field that would be scattered from it. The configuration corresponds to Fig. 1(b) with a stationary emitter.

The equivalent surface S_l is an ellipsoid enclosing the object as tightly as the geometry permits, so as to limit the extent of the control region, with the sources distributed at a spacing of $\lambda/5$. The simulations are conducted in air ($c = 344$ m/s), with all spatial dimensions normalized by the wavelength, the emitter radiating at $f = 6.88$ kHz, i.e. $\lambda = 5$ cm, with strength $\tilde{q}_e = 1$ kg/s². The two cylinders have radii of about 3λ and 2.7λ and lengths of about 7.8λ and 7.1λ , so that $kR \approx 17$ – 19 on their curved faces, well within the regime of PO validity away from the rims and the junction. The emitter is located on the ROI plane $z = 0$, which excludes the interior of S_l . The resulting fields are presented in Fig. 2(c)–(e).

Panel (c) shows the reference scattered field on the ROI, obtained by direct PO evaluation of the physical object using (5) with $\mathbf{r}_l$ replaced by the observation point $\mathbf{r}$. Panel (d) shows the reconstructed illusion field obtained from the control law (1)–(6), and panel (e) their difference. The accuracy is measured by the relative root mean square (rms) error, $e_{\text{rms}} = (\sum |p_{\text{syn}} - p_{\text{ref}}|^2 / \sum |p_{\text{ref}}|^2)^{1/2}$, with p_{ref} and p_{syn} the reference and synthesized fields and the sums running over the ROI points more than $\lambda/4$ outside S_l ; here $e_{\text{rms}} = 1.1\%$. Since both fields derive from the same PO approximation, this error measures the fidelity of the reconstruction, independently of the accuracy of PO itself.

The control signals are shown in Fig. 2(f), the dipole strengths being scaled by k so that both share the units kg/s². They are small, $|\tilde{q}^{(i)}|$ and $|k\tilde{F}^{(i)}|$ remaining below about 3×10^{-3} kg/s², some three orders of magnitude below $\tilde{q}_e$,

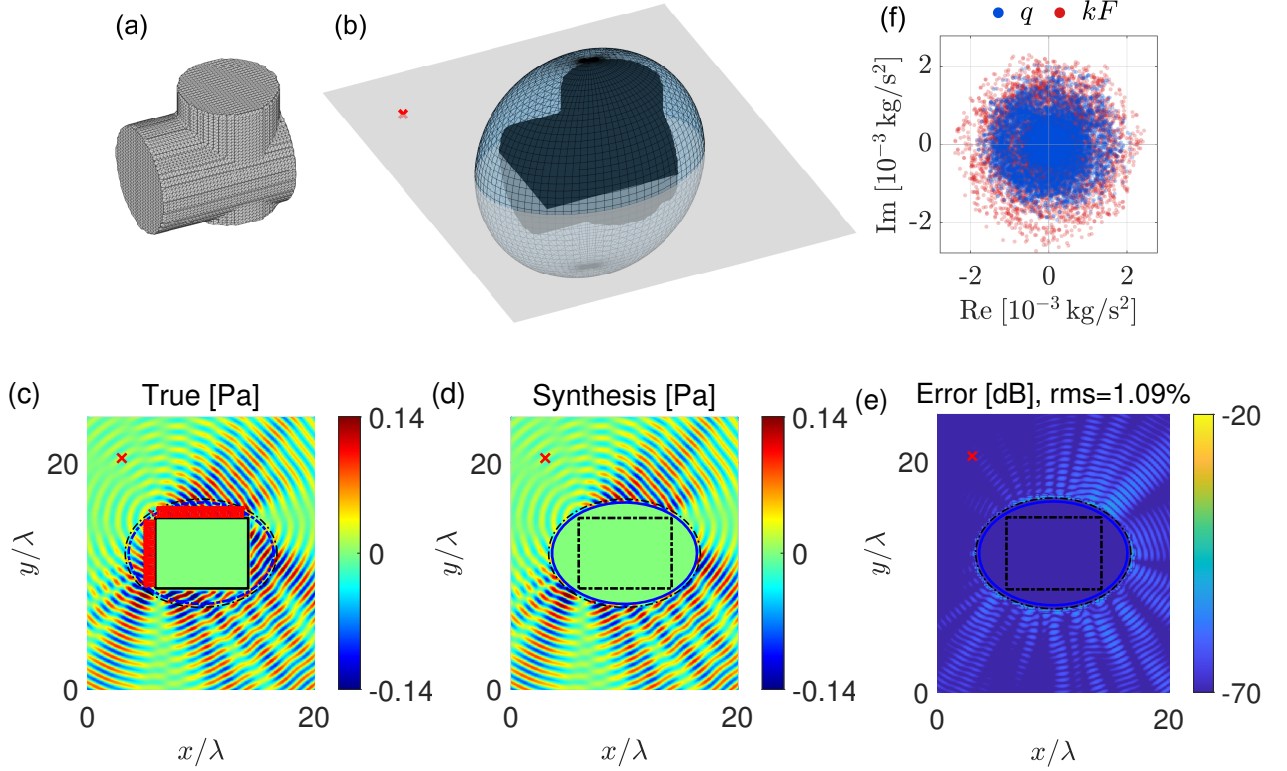


FIG. 2: Validation of the control algorithm in the frequency domain via numerical simulations based on (1)–(6). (a) The object. (b) The ellipsoidal equivalent surface S_l , with the object, the ROI plane and the emitter (red cross). (c)–(e) Top views of the ROI: the true scattered field, the synthesized illusion field and their difference in dB re 1 Pa, with the rms error in the title of (e). The black rectangle is the cross section of the object, with the outward normals at its illuminated points in red, and the blue curve is the intersection of S_l with the ROI; the field is not evaluated inside them and is shown as zero. (f) The control signals at all sources, in the complex plane.

since each source accounts only for the scattered field over its patch, the L contributions adding up in the ROI.

B. Time domain formulation

We now let the emitter move along a trajectory $\mathbf{r}_e(t_e)$ and radiate a possibly transient signal $q_e(t_e)$, so that no steady state is reached and the formulation must be carried out directly in the time domain. Defining $u_{\text{scat}} = \rho \dot{v}_{\text{scat}}$ as the counterpart of $\tilde{u}_{\text{scat}}$, the emulated scattered field at an observation point $\mathbf{r}$ exterior to S_l is expressed as a superposition of convolutional contributions from the control surface,

$$p_{\text{scat}}(\mathbf{r}, t) = \int_{S_l} \left[g^{p|q} * u_{\text{scat}} + g^{p|F} * p_{\text{scat}} \right](\mathbf{r}, t) dS_l, \quad (7)$$

the time-domain counterpart of (1). Here $*$ denotes temporal convolution, with the arguments of the convolved quantities suppressed for compactness. As in the harmonic case, the monopole and dipole layers are driven by the quantities on the equivalent surface itself, $u_{\text{scat}}(\mathbf{r}_l, t_l)$ and $p_{\text{scat}}(\mathbf{r}_l, t_l)$, related to the lumped strengths of a discrete array as in Sec. II A. Here the retarded time $t_l = t - R_l/c$ is the arrival time of the scattered field at $\mathbf{r}_l$, and also the emission time of the control source there.

The kernels $g^{p|q}$ and $g^{p|F}$ are the inverse transforms of (3a)

and (3b), built on the time-domain free-space Green's function

$$g(r, t) = \frac{\delta(t - r/c)}{4\pi r}, \quad (8)$$

the counterpart of (4). Since the control surface and the observation ROI are stationary, R_l is time-invariant, and with $j\omega \rightarrow \partial_{t_l}$ in α_l the convolutions in (7) collapse onto t_l ,

$$(g^{p|q} * u_{\text{scat}})(\mathbf{r}, t) = \frac{u_{\text{scat}}(\mathbf{r}_l, t_l)}{4\pi R_l}, \quad (9a)$$

$$(g^{p|F} * p_{\text{scat}})(\mathbf{r}, t) = \frac{\hat{\mathbf{R}}_l \cdot \hat{\mathbf{n}}_l}{4\pi c R_l} \left(\frac{\partial}{\partial t_l} + \frac{c}{R_l} \right) p_{\text{scat}}(\mathbf{r}_l, t_l). \quad (9b)$$

Equations (7)–(9) describe propagation from S_l to the ROI, which in a physical implementation is carried out by the medium itself. The control problem therefore reduces, as before, to determining p_{scat} and u_{scat} on S_l .

These are obtained here from the TDPO formulation for a moving emitter [28]. A contribution emitted at time t_e arrives at a point $\mathbf{r}'$ on the emulated object boundary at t' , and, upon reradiation, at a control point $\mathbf{r}_l$ at t_l , where

$$t' = t_e + \frac{R'_e(t_e)}{c}, \quad t_l = t' + \frac{R'_l}{c}, \quad (10)$$

with R'_e and R'_l the emitter-to-object and object-to- S_l distances of (6). The object and S_l being stationary, R'_l remains

time-invariant, whereas R'_e now varies with the emission time. Given t_l , the second relation therefore yields t' directly, while the first is implicit in t_e through $R'_e(t_e)$ and is inverted for it.

The scattered pressure generated by the emulated object at $\mathbf{r}_l \in S_l$ then follows as

$$p_{\text{scat}}(\mathbf{r}_l, t_l) = \frac{2}{4\pi} \int_{S_{\text{lit}}} \left[\frac{1}{c} \frac{\partial p_{\text{inc}}}{\partial t'} + \frac{p_{\text{inc}}}{R'_l} \right] \frac{\hat{\mathbf{R}}'_l \cdot \hat{\mathbf{n}}'}{R'_l} ds', \quad (11a)$$

the counterpart of (5), where the factor 2 is that of the PO surface field, and the bracket is evaluated at the incidence time $t' = t_l - R'_l/c$. The incident field at the surface point is that of a moving monopole [28],

$$p_{\text{inc}}(\mathbf{r}', t') = \frac{q_e(t_e)}{4\pi R'_e(t_e) [1 + \mu'_r(t_e)]}, \quad (11b)$$

where $\mu'_r(t_e) = c^{-1} \partial_{t_e} \mathbf{R}'_e \cdot \hat{\mathbf{R}}'_e = -\mathbf{v}_e \cdot \hat{\mathbf{R}}'_e/c$ is the directional Mach number of the emitter toward $\mathbf{r}'$, $\mathbf{v}_e = d\mathbf{r}_e/dt_e$ being its velocity, and $1/(1 + \mu'_r)$ the Doppler amplitude factor. Since $\partial t_e/\partial t' = 1/(1 + \mu'_r)$ by (10), the chain rule gives

$$\frac{\partial p_{\text{inc}}}{\partial t'} = \frac{1}{4\pi R'_e (1 + \mu'_r)^2} \left[\dot{q}_e - \frac{c q_e \mu'_r}{R'_e} - \frac{q_e}{1 + \mu'_r} \frac{\partial \mu'_r}{\partial t_e} \right], \quad (11c)$$

with $\partial \mu'_r/\partial t_e = c(\mu'^2 - \mu'^2_r)/R'_e - \mathbf{a}_e \cdot \hat{\mathbf{R}}'_e/c$, where $\mu = |\mathbf{v}_e|/c$ is the Mach number and $\mathbf{a}_e = d\mathbf{v}_e/dt_e$ the acceleration of the emitter. Here q_e , $\dot{q}_e$, R'_e and μ'_r are evaluated at the emission time t_e associated with t_l through (10), and the integration is again over the illuminated portion S_{lit} , defined by $\mathbf{R}'_e \cdot \hat{\mathbf{n}}' < 0$ at that emission time. The motion of the emitter thus enters only through $R'_e(t_e)$, $\mu'_r(t_e)$ and its derivative, evaluated along the prescribed trajectory. For a stationary emitter, $\mu'_r = 0$ and the retarded times in (10) become explicit. If the excitation is in addition harmonic, (11) reduces to (5) under the substitution $\dot{q}_e \rightarrow j\omega \tilde{q}_e$.

The numerical evaluation of (11) must contend with the implicit relation between the emission and incidence times in (10), handled by the two approaches termed *backward* and *forward* in [28], with S_l now taking the role of the observation surface. Only the path from the emitter to the object depends on the motion: the object and S_l being stationary, the path from $\mathbf{r}'$ to $\mathbf{r}_l$ contributes the fixed delay R'_l/c . With S discretized into cells centered at $\mathbf{r}'_m$, (11) is therefore evaluated in two stages. The first obtains p_{inc} and $\partial p_{\text{inc}}/\partial t'$ at each cell on the uniform incidence-time grid $t'_n = n\Delta t$, and the second sums the contributions of all cells at each control point $\mathbf{r}_l$, delayed by $R'_{l,m}/c$, with $R'_{l,m} = |\mathbf{r}_l - \mathbf{r}'_m|$.

In stage 1, the backward approach solves $t_e + R'_{e,m}(t_e)/c - t'_n = 0$ for t_e at each cell and grid sample, in closed form for simple trajectories and numerically otherwise. We adopt the forward approach, which solves no equation. The emission time is advanced uniformly, $t_e^{(i)} = i\Delta t$, and each sample is mapped to the incidence time $t_m^{(i)} = t_e^{(i)} + R'_{e,m}(t_e^{(i)})/c$, with $R'_{e,m}(t_e^{(i)}) = |\mathbf{r}'_m - \mathbf{r}_e(t_e^{(i)})|$. At these times, (11b) and (11c) are

evaluated, or set to zero if the cell is not illuminated at $t_e^{(i)}$, with $\dot{q}_e$, $\mathbf{v}_e$ and $\mathbf{a}_e$ taken as finite differences of the sampled signal and trajectory. Successive incidence times are spaced by $(1 + \mu'_{r,m})\Delta t$, with $\mu'_{r,m} = (R'_{e,m} - R'_{e,m}^{(i-1)})/(c\Delta t)$ the discrete counterpart of μ'_r toward the cell. The incidence times are thus compressed where the emitter approaches the cell and dilated where it recedes. For a subsonic emitter they increase monotonically, so that the cell signals on the uniform grid t'_n follow by interpolation, provided Δt resolves the shortest period received at the cells.

In stage 2, the delays are applied in the frequency domain, as the phase factors $e^{-j\omega R'_{l,m}/c}$ on the discrete Fourier transforms of the cell signals, so that $p_{\text{scat}}(\mathbf{r}_l, t_n)$ follows without a second interpolation in time. This uses the whole record, as available in the present simulations; in real time, the same delays may be applied sample by sample by interpolation, with fixed fractional delays. Both stages likewise give the pressures on the offset surfaces $S_l^\pm$, and, as in the frequency domain, $u_{\text{scat}}(\mathbf{r}_l, t_l) = -\partial_{n_l} p_{\text{scat}}$ follows by a finite difference between them, so that the control signals are now time series at each $\mathbf{r}_l$ rather than complex scalars.

The time-domain algorithm is demonstrated with the emulated object, control surface and emitter signal of Fig. 2, the emitter now moving along a trajectory in the ROI plane at Mach 0.2, as shown in Fig. 3(a). The control signals are evaluated from (11) by the forward approach, and the reconstruction follows from (7)–(9). Their peak magnitudes over the trajectory, $\max_t |q^{(i)}|$ and $\max_t |kF^{(i)}|$, are shown in Fig. 3(b), remaining about three orders of magnitude below the emitter amplitude, as in the harmonic case of Fig. 2(f).

Figure 3(c) compares snapshots of the reference field, obtained by direct TDPO evaluation for the physical object, with the synthesized field and their difference. As the emitter advances, the scattered pattern sweeps around the object, and the reconstruction follows it closely, with rms errors of 1.0–2.9%.

Control sources on an enclosing equivalent surface thus generate the illusion of a rigid object illuminated by a moving, possibly transient emitter. The enclosure requirement is relaxed next.

III. ILLUSION VIA AN INTERNAL CONTROL SURFACE

An enclosing control surface must be at least as large as the object itself, which becomes impractical as the object grows. We thus consider a control surface S_o that does not enclose the object, but is instead interior to it or covers it only partially.

The scattered field exists only outside the object, so it can be evaluated on S_l but not on S_o , and PO provides no continuation of it into the object. We therefore map the control signals on S_l , $\tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega)$ and $\tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega)$, onto S_o , a contracted version of S_l meshed at the same spacing, e.g. $\lambda/5$, and hence carrying fewer control points, $L_o < L$. The mapping is formulated in the frequency domain in Sec. III A, and in the time domain, where causality must be addressed, in Sec. III B.

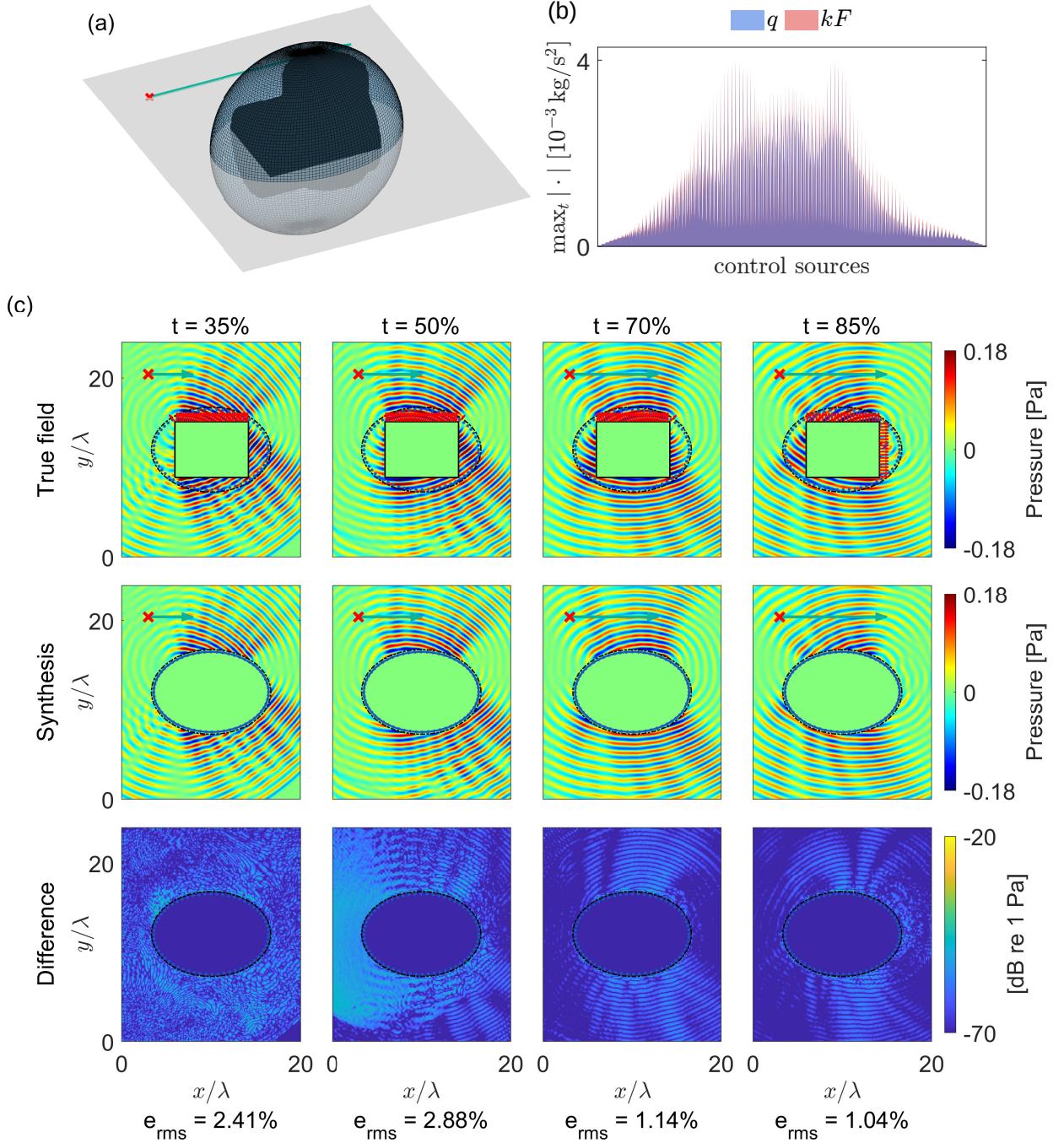


FIG. 3: Validation of the control algorithm in the time domain via numerical simulations based on (7)–(11). (a) Simulation setup, with the emitter trajectory (green arrow) starting at the red cross. (b) Peak magnitudes over time of the control signals at all sources. (c) Top views of the ROI at four instants, given as fractions of the trajectory duration: the true scattered field, the synthesized illusion field and their difference, with the rms error below each column. The black rectangle is the cross section of the object, with the outward normals at its illuminated points in red, and the blue curve is the intersection of S_l with the ROI; the field is not evaluated inside them and is shown as zero.

A. Frequency-domain formulation

We again employ the KH representation in (1), but now in a reverse (redatuming) sense, expressing the field on S_l in terms of equivalent source distributions defined on the internal surface S_o . In this context, points $\mathbf{r}_l$ on S_l , previously associated with control sources, are treated as observation points.

The source densities on S_o are written $\tilde{p}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)$ and $\tilde{u}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)$, in analogy with the pressure and scaled normal velocity driving the dipole and monopole layers on S_l , with $\hat{\mathbf{n}}_o$ the outward unit normal to S_o at $\mathbf{r}_o$. As on S_l , these are densities, in $\text{kg}/(\text{m}^2 \text{s}^2)$ and Pa, related to the lumped strengths of the redatumed array as in Sec. II A. The superscript marks them as dummy quantities: no scattered field exists on S_o , and

they are defined only by the requirement that they reproduce the control signals on S_l ,

$$\tilde{p}_{\text{scat}}(\mathbf{r}_l; \omega) = \int_{S_o} \left(G^{p|q} \tilde{u}_{\text{scat}}^{\text{eq}} + G^{p|F} \tilde{p}_{\text{scat}}^{\text{eq}} \right) dS_o, \quad (12a)$$

$$\tilde{u}_{\text{scat}}(\mathbf{r}_l; \omega) = \int_{S_o} \left(G^{u|q} \tilde{u}_{\text{scat}}^{\text{eq}} + G^{u|F} \tilde{p}_{\text{scat}}^{\text{eq}} \right) dS_o, \quad (12b)$$

where the arguments $(\mathbf{r}_o; \omega)$ of the source densities and $(R_{lo}; \omega)$ of the kernels are suppressed, with $\mathbf{R}_{lo} = \mathbf{r}_l - \mathbf{r}_o$ and $R_{lo} = |\mathbf{R}_{lo}|$. The kernels $G^{u|q}$ and $G^{u|F}$ give the scaled normal velocity at $\mathbf{r}_l$ radiated by a monopole and by a dipole at $\mathbf{r}_o$, and follow from $G^{p|q}$ and $G^{p|F}$ through $\tilde{u}_{\text{scat}} = -\partial_{n_l} \tilde{p}_{\text{scat}}$. Explicitly,

$$G^{p|q}(R_{lo}; \omega) = G(R_{lo}; \omega), \quad (13a)$$

$$G^{p|F}(R_{lo}; \omega) = G(R_{lo}; \omega) \alpha_{lo} \hat{\mathbf{R}}_{lo} \cdot \hat{\mathbf{n}}_o, \quad (13b)$$

$$G^{u|q}(R_{lo}; \omega) = G(R_{lo}; \omega) \alpha_{lo} \hat{\mathbf{R}}_{lo} \cdot \hat{\mathbf{n}}_l, \quad (13c)$$

$$G^{u|F}(R_{lo}; \omega) = -G(R_{lo}; \omega) \left(\beta \eta + \frac{\alpha_{lo} \hat{\mathbf{n}}_o \cdot \hat{\mathbf{n}}_l}{R_{lo}} \right), \quad (13d)$$

where $\hat{\mathbf{R}}_{lo} = \mathbf{R}_{lo}/R_{lo}$, $\alpha_{lo} = jk + R_{lo}^{-1}$, $\eta = (\hat{\mathbf{R}}_{lo} \cdot \hat{\mathbf{n}}_o)(\hat{\mathbf{R}}_{lo} \cdot \hat{\mathbf{n}}_l)$, and $\beta = k^2 - 3jkR_{lo}^{-1} - 3R_{lo}^{-2}$.

The left-hand sides of (12) are known, having been obtained from the scattering of the emulated object, so the unknowns are the two densities on S_o . Discretizing the surface integrals as sums over the L points of S_l and the L_o points of S_o gives the block system

$$\begin{bmatrix} \tilde{\mathbf{p}}_l \\ \tilde{\mathbf{u}}_l \end{bmatrix} = \begin{bmatrix} \mathbf{G}^{p|q} & \mathbf{G}^{p|F} \\ \mathbf{G}^{u|q} & \mathbf{G}^{u|F} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_o^{\text{eq}} \\ \tilde{\mathbf{p}}_o^{\text{eq}} \end{bmatrix}, \quad (14)$$

where $\tilde{\mathbf{p}}_l$ and $\tilde{\mathbf{u}}_l$ collect the data at the L points $\mathbf{r}_l^{(i)}$ of S_l , and $\tilde{\mathbf{u}}_o^{\text{eq}}$ and $\tilde{\mathbf{p}}_o^{\text{eq}}$ the densities at the L_o points $\mathbf{r}_o^{(m)}$ of S_o . Each block $\mathbf{G}^{\gamma|\delta} \in \mathbb{C}^{L \times L_o}$, $\gamma \in \{p, u\}$, $\delta \in \{q, F\}$, has entries $[\mathbf{G}^{\gamma|\delta}]_{i,m} = G^{\gamma|\delta}(R_{lo}^{(i,m)}; \omega)$. In the solution, the second block row of (14) is divided by $j\omega\rho$, so that the normal velocity $\tilde{v}_{\text{scat}}$ on S_l is fitted rather than $\tilde{u}_{\text{scat}}$. The kernels are otherwise unchanged and the unknowns remain the densities on S_o ; the scaling only sets the relative weight of the velocity and pressure data in the fit.

We write (14) as $\mathbf{b} = \mathbf{G}_{lo} \mathbf{x}$, with $\mathbf{b}$ the data vector, $\mathbf{G}_{lo}$ the block matrix and $\mathbf{x}$ the vector of unknowns. The system is ill-conditioned: propagation from S_o to the enclosing S_l attenuates the high spatial-frequency content of the densities, so the inverse amplifies it, together with any error in the data. Moreover, the monopole and dipole columns of $\mathbf{G}_{lo}$ differ widely in magnitude. We therefore solve in the general-form Tikhonov sense,

$$\mathbf{x}_\varepsilon = \left(\mathbf{G}_{lo}^\dagger \mathbf{G}_{lo} + \varepsilon^2 \mathbf{D}^2 \right)^{-1} \mathbf{G}_{lo}^\dagger \mathbf{b}, \quad (15)$$

where $\dagger$ denotes the Hermitian transpose, $\mathbf{D}$ is the diagonal matrix of the column norms of $\mathbf{G}_{lo}$, and $\varepsilon > 0$ is the regularization parameter. This is equivalent to scaling every column

of $\mathbf{G}_{lo}$ to unit norm and applying standard Tikhonov regularization to the scaled unknowns, so that ε is relative and each unknown is penalized in proportion to its influence on the data. Rather than through the normal equations, which would square the condition number, (15) is computed by QR factorization of the equivalent augmented least-squares system. The illusion pressure field at a point $\mathbf{r}$ in the ROI then follows by the same holography procedure as in (1), now applied on S_o ,

$$\tilde{p}_{\text{scat}}(\mathbf{r}; \omega) = \int_{S_o} \left(G^{p|q} \tilde{u}_{\text{scat}}^{\text{eq}} + G^{p|F} \tilde{p}_{\text{scat}}^{\text{eq}} \right) dS_o, \quad (16)$$

where the kernels are now evaluated at $R_o = |\mathbf{R}_o|$ with $\mathbf{R}_o = \mathbf{r} - \mathbf{r}_o$.

We repeat the frequency-domain simulation of Sec. II A for the same object and stationary emitter, now driving the contracted surface S_o rather than S_l . The control signals on S_l are those of Fig. 2, redatumed onto S_o by (15) with $\varepsilon = 10^{-3}$, S_o being S_l contracted by a factor of 0.6.

The scene is shown in Fig. 4(a), with the enclosing surface retained for reference. Panels (b)–(d) show the true, synthesized and difference fields, the synthesized field being evaluated from (16), and panel (e) the control signals on S_o .

Agreement is assessed outside S_l only, since the redatumed densities are fitted to reproduce the control signals on S_l and the representation carries no accuracy guarantee in the region between the two surfaces. Outside S_l the rms error is 2.8%, against 1.1% with the enclosing surface of Sec. II A, the increase reflecting the regularized inversion.

B. Time-domain formulation

For a stationary emitter radiating a harmonic signal, the time-domain control signals follow directly from the densities obtained in Sec. III A. With the convention of Sec. II A, the sources on S_o are driven by

$$p_{\text{scat}}^{\text{eq}}(\mathbf{r}_o, t) = |\tilde{p}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)| \cos[\omega t + \arg \tilde{p}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)], \quad (17a)$$

$$u_{\text{scat}}^{\text{eq}}(\mathbf{r}_o, t) = |\tilde{u}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)| \cos[\omega t + \arg \tilde{u}_{\text{scat}}^{\text{eq}}(\mathbf{r}_o; \omega)], \quad (17b)$$

each point of S_o carrying a sinusoid of the same frequency, with amplitude and phase set by the redatumed solution.

Causality now enters. On the enclosing S_l the control signals coincide with the scattered field and are causal by construction. The sources on S_o lie inside S_l , so their radiation must leave S_o ahead of the field due on S_l , by R_{lo}/c for a pair of points and by at most $T_o = \max_{\mathbf{r}_l \in S_l, \mathbf{r}_o \in S_o} R_{lo}/c$. Each source on S_o is therefore switched on at the latest instant that still reaches all of S_l in time, $t_o^{\text{on}} = \min_{\mathbf{r}_l \in S_l} (t_l^a - R_{lo}/c)$, with t_l^a the first arrival of the scattered field at $\mathbf{r}_l$. Where this would precede the emission, the source starts at the emission instant and reaches part of S_l late.

To limit the switch-on transient, each source is also faded in over two periods by the raised-cosine window $w(t) = \{1 -$

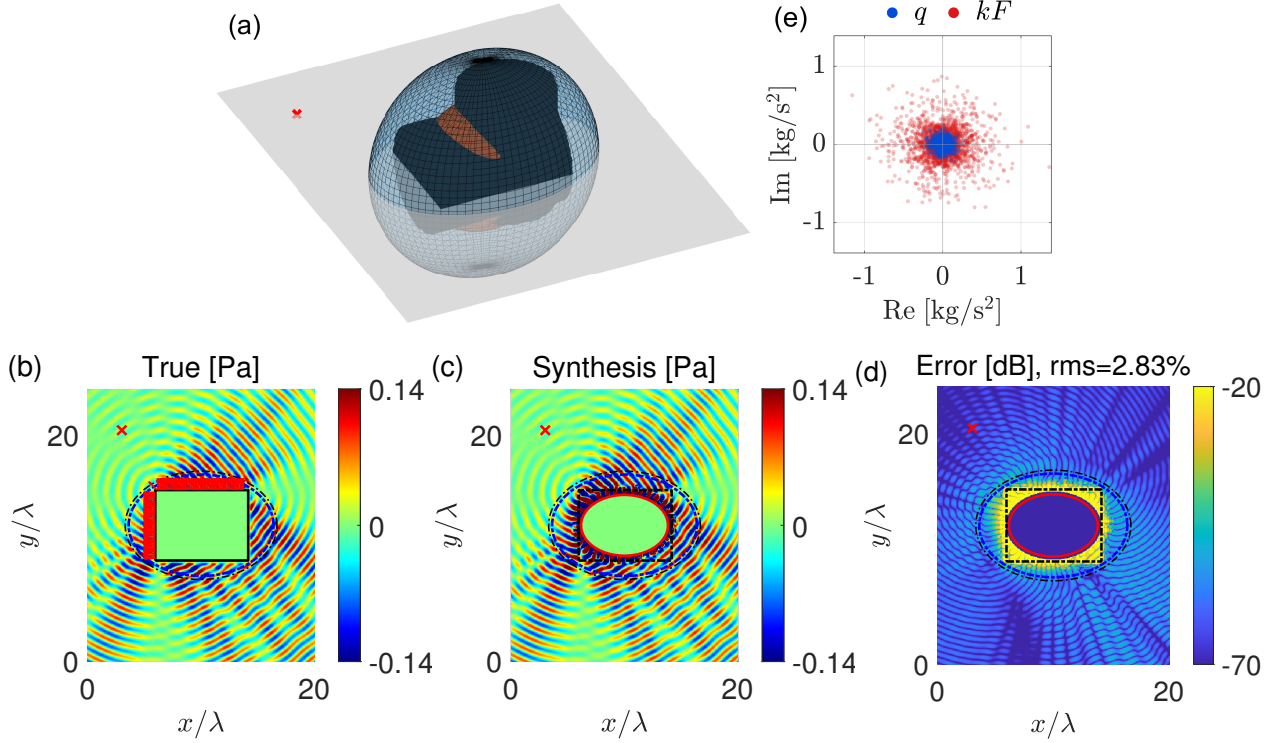


FIG. 4: Illusion via an internal control surface, frequency domain. (a) Scene: the internal control surface S_o (red) carrying the redatumed control sources, the enclosing surface S_l (blue) shown for reference, and the stationary emitter (red cross). (b) Scattered pressure field of the physical object, (c) field synthesized by the sources on S_o , and (d) their difference. The error is assessed outside S_l , the region between S_o and S_l being excluded. (e) The control signals at all sources on S_o in the complex plane.

$\cos[\pi(t - t_o^{\text{on}})/T_r]\}/2$ for $0 \leq t - t_o^{\text{on}} \leq T_r = 4\pi/\omega$, with $w = 0$ before and $w = 1$ after. In the harmonic steady state neither measure costs accuracy: once every source is fully on, the synthesized field is the same as without them, the advance being contained in the phases of (17).

The densities on S_o are obtained as in Sec. III A, with the same ϵ , from the complex amplitudes of the TDPO signals on S_l , extracted by a harmonic fit over their last three periods, and are converted to the sinusoids (17). The results, for the stationary emitter of Fig. 4, are shown in Fig. 5. Here $T_o \approx 1.5$ ms, and the switch-on instants range from the emission instant to 0.71 ms. The peak control signals are shown in Fig. 5(b). Figure 5(a) compares snapshots of the reference, synthesized and difference fields, the error being assessed outside S_l as in Sec. II B. At 21% of the duration the synthesized field is still building up and departs strongly from the reference; by 41% the error has dropped to 4.4%, and it settles at about 3% once all sources are fully on and the initial transient has passed, Fig. 5(c).

IV. CONCLUSION

In this work, we created a three-dimensional acoustic illusion of a rigid object illuminated by a moving emitter, using control sources on an equivalent surface. The sources, monopoles and dipoles, were driven by the field the object would scatter onto the surface, their signals following in

closed form from a time-dependent physical optics formulation that we developed earlier for a moving emitter [28] and recapped here in (10)-(11).

Because the object surface is treated as locally flat and rigid, the scattered field follows by direct integration over it, and the motion only shifts the times at which each of its points is illuminated. The scattering problem is thus not re-solved at each emitter position, nor is the propagation domain discretized. The control algorithms were tested in simulations on a non-convex object composed of two intersecting cylinders. With an enclosing surface, the rms error was 1.1% for a stationary emitter and 1.0–2.9% at Mach 0.2. The control signals remained three orders of magnitude below the emitter strength, the effort being distributed over many sources, whose number is in turn set by the spacing that a desired level of accuracy requires (here set to $\lambda/5$).

For objects too large to enclose, we proposed a mapping of the control signals onto a smaller, internal surface that lies inside, or partly inside, the region the object would occupy and, at the same spacing, carries fewer sources. The field is then reproduced only outside the enclosing surface, where, for a stationary emitter, the rms error was 2.8% in the frequency domain and about 3% in the time domain, of the same order of magnitude as with the enclosing surface. The penalties are a regularized inversion in place of the closed form, control signals more than two orders of magnitude larger, Figs. 2(f) and 4(e), and a start-up transient wherever causality would require a source to radiate before the emission. A moving

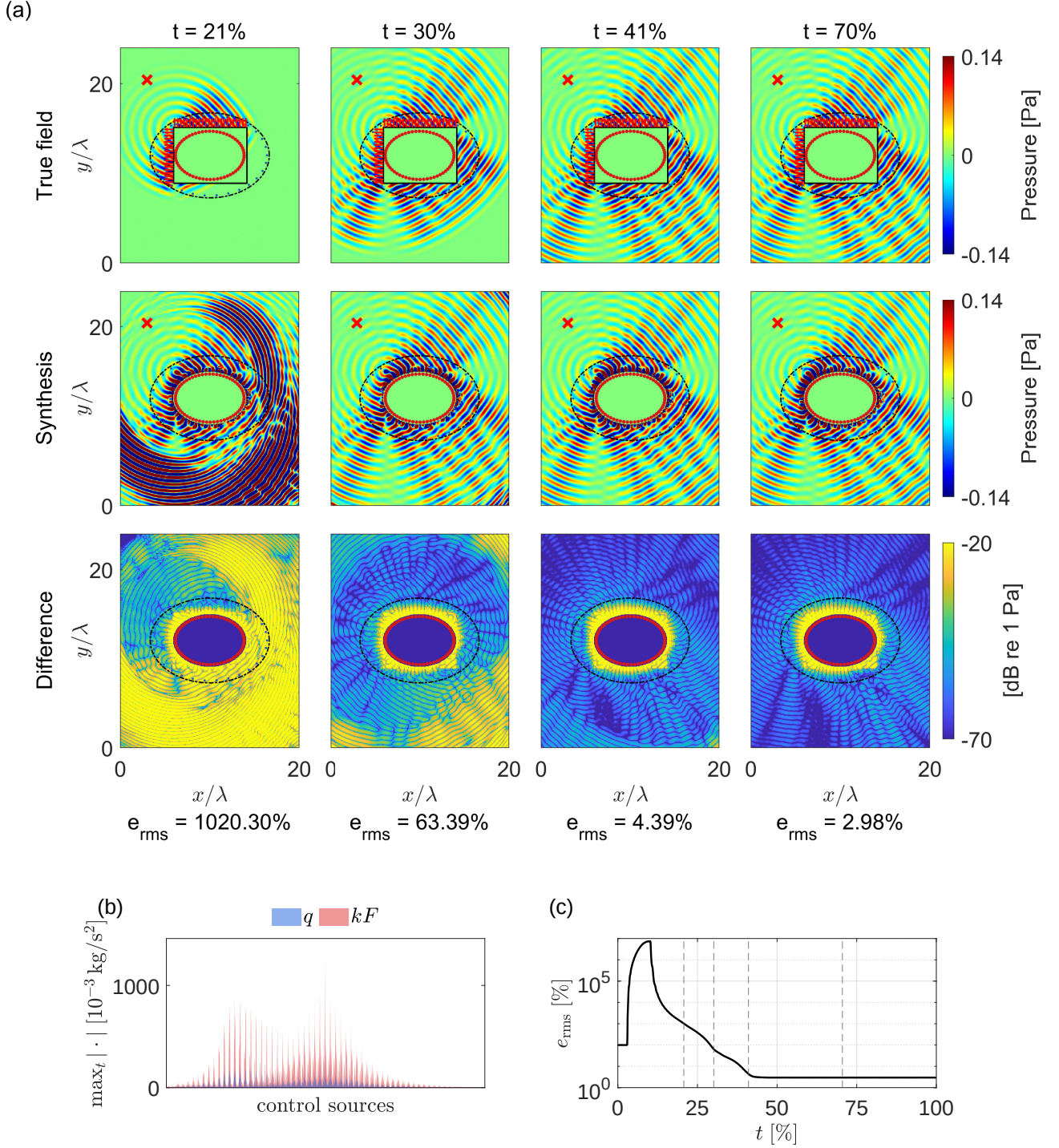


FIG. 5: Time-domain illusion via an internal control surface, for the stationary emitter of Fig. 4, radiating at $f = 6.88$ kHz ($\lambda = 5$ cm), with the sources on S_o spaced by $\lambda/5$, as on S_l . (a) Top views of the ROI at four instants chosen across the start-up, given as fractions of the simulation duration of 10.2 ms, with the rms error below each column. S_o is shown in red, S_l dashed and the emitter by the red cross; the error is assessed outside S_l . (b) Peak magnitudes over time of the control signals at all sources. (c) Evolution of the rms error, the dashed lines marking the instants of (a).

emitter was not considered in this configuration.

Reference and illusion derive from the same PO fields, so these errors measure the reconstruction, not PO itself. PO is approximate at the rims, and at the junction of the cylinders, where multiple scattering between them is neglected.

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