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A classical-mechanical analogue of Hawking black hole radiation

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Abstract – **One of the most fascinating phenomena in general relativity is the black hole, and the associated Hawking radiation, which predicts that escape from the hole can be possible for relativistic particles under certain conditions. This phenomenon has been recently highlighted by a quantum condensed matter analogue, using a microscopic lattice model of a Weyl semimetal with tilted nodes. Here, we are motivated to imitate black hole Hawking radiation in a purely classical-mechanical system. We propose a coupled double chain model featuring frequency dispersion that coincides with the tilted Weyl semimetal bandstructure near the vertices. The resulting directional inter-site couplings can be realized by an active mechanical metamaterial with an underlying programmable feedback network.**

I. INTRODUCTION

The idea to guide classical waves by mimicking quantum-mechanical wave phenomena has received a major interest in recent years. The underlying idea is based on the direct analogy between the electronic bandstructure of solids and the frequency dispersion of classical systems [1], and enables to transfer ideas from quantum condensed matter systems into the classical realm. For example, a great deal of attention was devoted to mimicking quantum topological phenomena, such as the quantum Hall, the quantum spin-Hall or the quantum valley-Hall effects [2, 3].

However, an entire class of quantum-mechanical phenomena related to *tunneling* remains under-explored for classical waveguiding. These phenomena includes, e.g., Klein tunneling of relativistic particles through potential barriers of arbitrary heights and widths [4], tunneling of particles across the event horizon of black and white holes [5], tunneling of electron pairs through superconducting junctions [6], and more. The common property of these effects, which constitutes the essence of tunneling, is an unusual and counter-intuitive ability of particles to cross gaps, barriers or interfaces, despite this crossing being seemingly forbidden by energy considerations. Obtaining a purely classical realization of these properties could substantially advance waveguiding capabilities in classical systems, and therefore serves the motivation of this research.

Here we focus at the black hole horizon tunneling. The creation of a black hole is a result of compression of a massive object to the point where light is prevented to escape from its vicinity due to gravity. The hole region is limited by the event horizon. This is the distance from the hole below which no escape is possible [7]. However, Hawking showed that for relativistic quantum effects, such as spontaneous creation of particle-antiparticle pairs close to the event horizon, an escape (tunneling) from the black hole is possible for one partner of the pair.

Recently, an analogy between the band-structure of Weyl semimetals with inhomogenously tilted nodes and the spacetime metric of black holes has been formally derived [8–11], suggesting that the tilted Weyl nodes exhibit properties of Hawking radiation. A particular tilt, denoted by the critical tilt, represents the event horizon.

Here we propose a classical metamaterial model, consisting of two parallel cross-coupled chains. We study the associated spectral characteristics, and demonstrate a striking similarity between the metamaterial frequency dispersion and the condensed matter system energy dispersion at the vicinity of its singular points.

II. THE QUANTUM CONDENSED MATTER MODEL

The schematics in Fig.1(a) represents a one-dimensional propagation path along the principal axis of a Weyl semimetal system, which captures the condensed matter model of horizon tunneling. In momentum space, the path

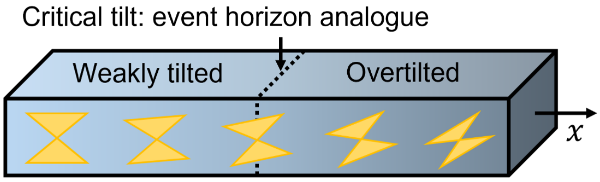

Fig. 1: Schematic of black hole horizon tunneling analogue in a Weyl semimetal with spatially-varying tilt.

features Weyl cones (gold) with a space-dependent tilt [11], characterized by the quantum Hamiltonian H_q,

$$H_q(k) = -\hat{t}_1 \sin ka \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + t_1(1-\cos ka) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - V(x_0) \sin ka \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (1)$$

Here t_1 and $\hat{t}_1$ are constant and positive parameters representing electron hopping. $V(x_0)$ is the potential that creates the required tilt in the electronic dispersion cones with a spatial dependence of $V(x) = -(1+\tanh \alpha x)$, where α is a constant. The potential is calculated at a location of interest x_0 along the interface $-L \leq x \leq L$, where $x = 0$ is the critical tilt indicating the event horizon analogue, and L is big enough so that $\tanh(L) \approx 1$. Outside the interface, i.e. at $x < -L$ and $x > L$, the potential takes the constant end values of the $V(x)$ function.

III. The classical analogue model

To derive the analogous classical-mechanical model of Eq.(1), we consider a double mass-spring chain of periodicity a, consisting of masses at two inequivalent sites A (top chain) and B (bottom chain). The structure in Eq. (1) implies that there are four kind of couplings, $+t_1$, $-t_1$, $\hat{t}_1$ and $V(x)$. The A sites are coupled by real and positive elements, analogous to linear springs of stiffness $+t_1$. The B sites are coupled by negative elements, analogous to unstable springs of stiffness $-t_1$. Both A and B are coupled to the adjacent sites by complex-valued elements $i\hat{t}_1$ and $iV(x)$, in cross and direct paths, respectively.

The $-t_1$ couplings make the total classical system dynamically unstable, corresponding to the existence of imaginary spectrum. To reach the traveling-harmonic solution, the system needs to be stabilized, but in such a way that the tunneling properties are preserved. A possible technique, as confirmed by the results below, is adding a local positive coupling β to both A and B sites. This can be regarded as an additional stable spring connecting each mass to the ground. We found that $\beta = 4t_1$ is valid, leading to the adapted Hamiltonian $H_c(\mathbf{k}) = H_q(\mathbf{k}) + 4t_1 I$.

To implement the complex-valued couplings $i\hat{t}_1$ and $iV(x)$, we invoke the fact that the inverse Fourier transform of iu equals to $\dot{u}/\omega_0$ or v/ω_0, where v is the out-of-plane velocity of the masses, and ω_0 is the working frequency. This leads to the classical model

$$\begin{cases} \ddot{u}_m^A &= \hat{t}_1 \frac{1}{2\omega_0}\left(-v_{m+1}^B + v_{m-1}^B\right) + t_1 \frac{1}{2}\left(-2u_m^A + u_{m+1}^A + u_{m-1}^A\right) + V(x_m)\frac{1}{2\omega_0}\left(-v_{m+1}^A + v_{m-1}^A\right) - \beta u_m^A, \\ \ddot{u}_m^B &= \hat{t}_1 \frac{1}{2\omega_0}\left(-v_{m+1}^A + v_{m-1}^A\right) - t_1 \frac{1}{2}\left(-2u_m^B + u_{m+1}^B + u_{m-1}^B\right) + V(x_m)\frac{1}{2\omega_0}\left(-v_{m+1}^B + v_{m-1}^B\right) - \beta u_m^B, \end{cases} \quad (2)$$

in which the $i\hat{t}_1$ and $iV(x)$ couplings are achieved in steady-state. However, the complex value of the couplings is not as much of a problem, as their single-sided/directional nature, i.e. not involving a restoring term $v_m^{A,B}$, and thus not being directly supported by Newtonian dynamics. Their (and the $-t_1$ unstable terms) realization can be carried out by metamaterials with an embedded feedback mechanism [12–14], in which the non-physical couplings can be programmed into controllers, and obtained in steady-state in a real-time closed-loop operation.

We now demonstrate the frequency domain validation of the model in Eq.(2). In Fig. 2 we plot the quantum energy dispersion of the tilted Weyl cones versus the classical frequency dispersion at three different locations along the black hole interface: to the left of the interface with $V = 0$, to its right with $V = -2$, and in the middle, corresponding to the critical tilt at $x = 0$ with $V = -1$. We observe that despite the profound difference between the underlying physics of both models, their spectrum shape coincides throughout the entire propagation path.

IV. Conclusion

We presented a mechanism to realize black hole Hawking radiation in classical metamaterials. We showed that the classical and the quantum dispersions quantitatively match.

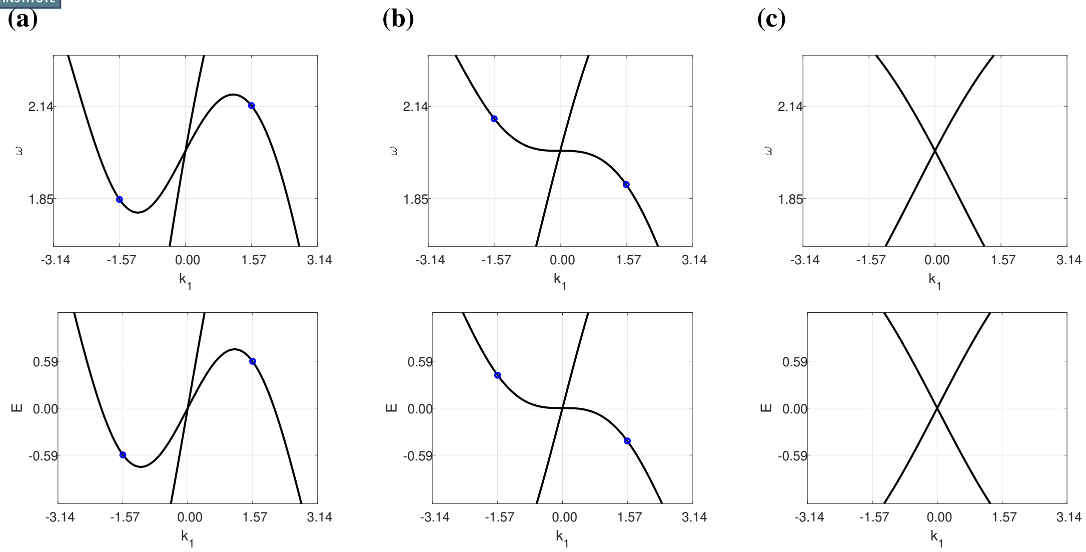

Fig. 2: Classical (top) vs. quantum (bottom) spectrum of horizon tunneling, at $V = -2$ (a), $V = -1$ (b), and $V = 0$ (c).

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References

[1] M. Franz and L. Molenkamp, *Topological Insulators*. Elsevier, 2013.

[2] F. D. M. Haldane, "Model for a quantum Hall effect without Landau levels: Condensed-matter realization of the "parity anomaly"," *Physical Review Letters*, vol. 61, no. 18, p. 2015, 1988.

[3] C. L. Kane and E. J. Mele, "Quantum spin Hall effect in graphene," *Physical Review Letters*, vol. 95, no. 22, p. 226801, 2005.

[4] M. Katsnelson, K. Novoselov, and A. Geim, "Chiral tunnelling and the Klein paradox in graphene," *Nature Physics*, vol. 2, no. 9, pp. 620–625, 2006.

[5] S. W. Hawking, "Particle creation by black holes," in *Euclidean quantum gravity*. World Scientific, 1975, pp. 167–188.

[6] R. F. Voss and R. A. Webb, "Macroscopic quantum tunneling in 1-μm nb Josephson junctions," *Physical Review Letters*, vol. 47, no. 4, p. 265, 1981.

[7] G. E. Volovik, "Black hole and Hawking radiation by type-II Weyl fermions," *JETP Letters*, vol. 104, no. 9, pp. 645–648, 2016.

[8] G. Volovik and M. Zubkov, "Emergent Weyl spinors in multi-fermion systems," *Nuclear Physics B*, vol. 881, pp. 514–538, 2014.

[9] Y. Xu, F. Zhang, and C. Zhang, "Structured Weyl points in spin-orbit coupled fermionic superfluids," *Physical Review Letters*, vol. 115, no. 26, p. 265304, 2015.

[10] A. A. Soluyanov, D. Gresch, Z. Wang, Q. Wu, M. Troyer, X. Dai, and B. A. Bernevig, "Type-II Weyl semimetals," *Nature*, vol. 527, no. 7579, pp. 495–498, 2015.

[11] D. Sabsovich, P. Wunderlich, V. Fleurov, D. I. Pikulin, R. Ilan, and T. Meng, "Hawking fragmentation and Hawking attenuation in Weyl semimetals," *Physical Review Research*, vol. 4, no. 1, p. 013055, 2022.

[12] L. Sirota, R. Ilan, Y. Shokef, and Y. Lahini, "Non-Newtonian topological mechanical metamaterials using feedback control," *Physical Review Letters*, vol. 125, no. 25, p. 256802, 2020.

[13] L. Sirota, Y. Lahini, R. Ilan, and Y. Shokef, "Feedback-based topological mechanical metamaterials," in *Fourteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials)*. IEEE, 2020, pp. 415–417.

[14] L. Sirota, D. Sabsovich, Y. Lahini, R. Ilan, and Y. Shokef, "Real-time steering of curved sound beams in a feedback-based topological acoustic metamaterial," *Mechanical Systems and Signal Processing*, vol. 153, p. 107479, 2021.