Double-Negative Temporal Acoustic Metamaterials

O. Lasri[1] **and L. Sirota**[1]
[1]Tel Aviv University, School of Mechanical Engineering, Tel Aviv 69978, Israel
orlasri@mail.tau.ac.il, leabeilkin@tauex.tau.ac.il

Abstract **– Metamaterials with temporal modulation of constitutive parameters have recently emerged to the scientific spotlight, featuring the novel concept of temporal boundaries. These boundaries, which are artificial by spatial means, cause in-domain wave reflection and transmission at a temporal frequency different from that of the source, in contrast to the case of spatial boundaries or interfaces. Here we study a new form of temporal modulation, which drives the system from a dispersionless to a dispersive state in both constitutive parameters, supporting the so-called Drude model. We demonstrate that the resulting temporal boundary generates waves simultaneously in two different dynamical regimes, double-negative and double-positive, for a single frequency source, which is an otherwise unachievable response.**

I. Introduction

Artificially manufactured structures, termed metamaterials, enable advanced wave propagation, breaking barriers in wave manipulation and control. A celebrated example is the zero or negative material parameters property, which enables dynamical behaviors unachievable in conventional materials [1–3]. Applications of metamaterials with space or frequency modulated parameters have been extensively addressed over the last two decades, including invisibility cloaking [4], classical analogues of quantum wave phenomena [5–8], and many more [9].

Recently, the new class of time-modulated metamaterials has gained an increased interest. Concepts such as Fresnel coefficients, Snell's law [10] and Brewster angle [11] have been developed analogously for the case of time-varying systems, as well as anti-reflection temporal coatings [12], temporal aiming [13] and other techniques for wave manipulation.

These concepts, however, were carried out either for a dispersionless case, where the constitutive parameters are constant, and a pair of frequency-shifted forward and backward waves are generated, or for a situation in which a single constitutive parameter is frequency-dependent [14], yielding two pairs of waves with shifted frequencies.

However, the property of double-negative medium has not been researched in time-modulated metamaterials. Here, we derive the analytical solution for a temporally-varying unbounded acoustic medium with a step-like change both in the effective mass density and bulk modulus, from constant to frequency-dependent values that support the double-negative regime.

II. System Description

The schematic in Fig 1 illustrates the proposed system, a fluid channel of static uniform mass density ρ_0 and bulk modulus b_0. The channel is assumed to be infinite, a feature which can be achieved with perfectly absorbing boundary conditions [15]. An acoustic velocity source ψ of frequency ω_0 drives the system at $x = 0, t = t_0$. We denote the interval $t_0 \leq t < t_s$ as medium I, where the effective medium parameters remain constant.

In order to investigate new capabilities of wave manipulation under temporal variation, we consider metamaterials with single zero (SZ), single negative (SN), double zero (DZ) and double negative (DN) parameters, which are commonly explored in spatially-modulated systems. We introduce a temporal discontinuity to the system at $t = t_s$, through which the medium effective properties are rapidly changed from the constant uniform values to frequency-dependent values, according to the well-known Drude Model

$$\tilde{M}(\omega) = \frac{\omega^2 - \omega_m^2}{\omega^2} \qquad , \qquad \tilde{B}(\omega) = \frac{\omega^2}{\omega^2 - \omega_\beta^2}, \tag{1}$$

where $\omega_m = m\omega_0$ and $\omega_\beta = \beta\omega_0$ for constant and positive m and β.

III. Results

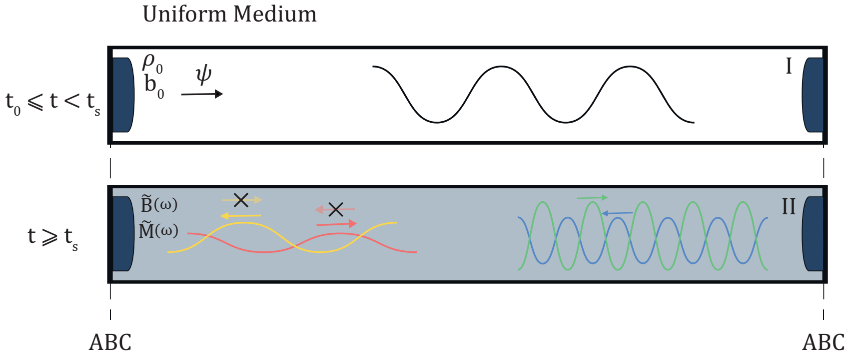

Fig. 1: System description. top: uniform medium before the temporal switch. bottom: dispersive medium parameters, after the temporal switch.

We denote the interval $t \geq t_s$ as medium II. The only frequencies supported by the medium are those that satisfy momentum conservation with the wavenumber $k = \frac{\omega_0}{c}\sqrt{\tilde{M}(\omega)\tilde{B}^{-1}(\omega)}$. We therefore calculate the frequncies who conserve momentum ω_i

$$\frac{\omega_0}{c} = \frac{\omega_i}{c}\sqrt{\tilde{M}(\omega_i)\tilde{B}^{-1}(\omega_i)} \tag{2}$$

(2) yields the following equation

$$\omega_i^4 - (\omega_0^2 + \omega_m^2 + \omega_\beta^2)\omega_i^2 + \omega_m^2\omega_\beta^2 = 0 \tag{3}$$

The generation of two pairs of waves can be therefore seen in the solution of (3), which consists of two different frequency values, each belongs to both waves in a single pair,

$$\omega_i = \frac{1}{\sqrt{2}}\left(\omega_0^2 + \omega_m^2 + \omega_\beta^2 \pm \left((\omega_0^2 + \omega_m^2 + \omega_\beta^2)^2 - 4\omega_m^2\omega_\beta^2\right)^{1/2}\right)^{1/2}, \qquad i = 1, 2. \tag{4}$$

$\pm\omega_i,\ i = 1, 2$ are the frequencies of the waves generated after the temporal switch.

Fig. 2: Division of dynamical regimes with respect to frequency

Assuming that the frequency solutions in (4) are given in ascending order, it can be seen in Fig. 2 that the larger solution propagates through a double positive (DP) regime while the smaller propagates through the DN regime of the medium. The resulting behavior of each pair generated by the temporal discontinuity is different. In particular, the pair of the larger frequency propagates with both phase and group velocities being positive, while the other propagates with negative group velocity and positive phase velocity. This in turn leads to the propagation of the phase fronts and energies in reverse directions.

IV. Conclusion

We studied the effect of a step-like change in the effective parameters of an acoustic medium, from constant to dispersive values in both constituitve parameters. The resulting waves generated by the temporal boundary propagate through a new dynamical regime: one pair in the double-positive regime and another in the double-negative regime.

References

[1] J. Li and C. Chan, "Double-negative acoustic metamaterial," *Physical Review E*, vol. 70, no. 5, p. 055602, 2004.

[2] I. Liberal and N. Engheta, "Near-zero refractive index photonics," *Nature Photonics*, vol. 11, no. 3, pp. 149 – 58, 2017/03/.

[3] M. Dubois, C. Shi, X. Zhu, Y. Wang, and X. Zhang, "Observation of acoustic dirac-like cone and double zero refractive index," *Nature Communications*, vol. 8, p. 14871, 2017.

[4] S. A. Cummer and D. Schurig, "One path to acoustic cloaking," *New Journal of Physics*, vol. 9, no. 3, p. 45, 2007.

[5] S. H. Mousavi, A. B. Khanikaev, and Z. Wang, "Topologically protected elastic waves in phononic metamaterials," *Nature Communications*, vol. 6, p. 8682, 2015.

[6] S. Yves, R. Fleury, F. Lemoult, M. Fink, and G. Lerosey, "Topological acoustic polaritons: robust sound manipulation at the subwavelength scale," *New Journal of Physics*, vol. 19, no. 7, p. 075003, 2017.

[7] L. Sirota, R. Ilan, Y. Shokef, and Y. Lahini, "Non-Newtonian topological mechanical metamaterials using feedback control," *Physical Review Letters*, vol. 125, no. 25, p. 256802, 2020.

[8] L. Sirota, D. Sabsovich, Y. Lahini, R. Ilan, and Y. Shokef, "Real-time steering of curved sound beams in a feedback-based topological acoustic metamaterial," *Mechanical Systems and Signal Processing, in press*, 2020.

[9] S. A. Cummer, J. Christensen, and A. Alù, "Controlling sound with acoustic metamaterials," *Nature Reviews Materials*, vol. 1, no. 3, pp. 1–13, 2016.

[10] J. Mendonça and P. Shukla, "Time refraction and time reflection: two basic concepts," *Physica Scripta*, vol. 65, no. 2, p. 160, 2002.

[11] V. Pacheco-Peña and N. Engheta, "Temporal equivalent of the brewster angle," *Phys. Rev. B*, vol. 104, p. 214308, Dec 2021.

[12] V. Pacheco-Peña and N. Engheta, "Antireflection temporal coatings," *Optica*, vol. 7, no. 4, pp. 323–331, 2020.

[13] ——, "Temporal aiming," *Light: Science & Applications*, vol. 9, no. 1, pp. 1–12, 2020.

[14] D. M. Solís, R. Kastner, and N. Engheta, "Time-varying materials in the presence of dispersion: plane-wave propagation in a lorentzian medium with temporal discontinuity," *Photonics Research*, vol. 9, no. 9, pp. 1842–1853, 2021.

[15] L. Sirota and A. M. Annaswamy, "Active boundary and interior absorbers for one-dimensional wave propagation: Application to transmission-line metamaterials," *Automatica*, vol. 117, pp. 108–855, 2020.